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Measurement of Dynamic Efficiency in Production: An Application of Data Envelopment Analysis to Japanese Electric Utilities

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Abstract

The purpose of this paper is to measure productive efficiencies when a firm employs quasi-fixed inputs that cannot be instantaneously adjusted to their optimal levels. To this end, data envelopment analysis (DEA) is extended to a dynamic framework so that investment behavior can be modelled with the efficient production frontier. Based on the work of Nemoto and Goto (1999), we show how the efficiencies of quasi-fixed inputs and their adjustment processes are evaluated. An application to Japanese electric utilities over the 1981–1995 period delivers empirically plausible results and proves the usefulness of the procedure.

JEL classification: C61, D92, L94

Keywords: data envelopment analysis, dynamic optimization, dynamic efficiency, Hamilton-Jacobi-Bellman equation

1. Introduction

The purpose of this paper is to measure productive efficiencies when a firm employs quasi-fixed inputs that cannot be instantaneously adjusted to their optimal levels. To this end, data envelopment analysis (DEA) is extended to a dynamic framework so that investment behavior can be modelled with the efficient production frontier. Based on the work of Nemoto and Goto (1999), we show how the efficiencies of quasi-fixed inputs and their adjustment processes are evaluated. An application to Japanese electric utilities over the 1981–1995 period delivers empirically plausible results and proves the usefulness of the procedure.

Since the pioneering work of Farrell (1957), production efficiency has been measured as the distance between an observation and an estimated ideal referred to as an efficient

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frontier. Over the last two decades, a number of econometric and DEA techniques have been developed to estimate the efficient frontier in a way consistent with the economic theory of optimizing behavior of a firm. However, except for a few studies in the DEA literature, most previous works have stayed within a static framework and failed to model the intertemporal behavior of a firm. To the best of our knowledge, only Sengupta (1995) and Färe and Grosskopf (1996) have introduced some dynamic aspects of production into their DEA models.

Färe and Grosskopf formulate several kinds of intertemporal substitution in the form of multiperiod linear programming (LP) problems, which describe more realistic production processes than are treated by the static DEA. While their primary interests are storable inputs and intermediate outputs, their network model is adaptable for analyzing the behavior of investment in quasi-fixed inputs. Nevertheless, they do not explicitly state the conditions for the optimal paths of adjustment that play a central role in investment theory. For example, Färe and Grosskopf (1997) incorporate endogenous investment into the network model so as to evaluate the performance of economic growth for countries in the Asian-Pacific Economic Community. However, the optimality conditions for investment are not discussed because their primary interest lies in the radial measure of efficiency and formulate neither cost minimization nor profit maximization as an objective.

On the other hand, Sengupta highlights the importance of the first order conditions of intertemporal optimization in constructing a dynamic DEA. He suggests introducing those conditions to a set of constraints in the analytic LP problem. However, his approach does not necessarily clarify the internal relationship between a firm's behavior and its inefficiency because it is still implicit in the theoretical foundation of a behavioral assumption about a firm. In this sense, the first order conditions given by Sengupta's model are insightful but not fully interpretable in an economic perspective.

Recently, extending the previous works, Nemoto and Goto (1999) developed a more comprehensive and practicable procedure.[1] They formulated an analytic LP problem from which the optimality conditions are explicitly derived as a result of the LP duality theorem. As a result, their procedure is closely related to the adjustment-cost theory of investment, so that it provides a nonparametric alternative to the econometric Euler equation approach.

In this paper, we empirically implement Nemoto and Goto's procedure for the first time. This paper furthers previous studies in two ways. Firstly, it presents a measurement scheme of dynamic efficiencies in quasi-fixed inputs and their adjustment processes. Secondly, it shows how investment behavior can be modelled with a DEA technique. A main idea in Nemoto and Goto's procedure is to augment conventional DEA by treating quasi-fixed inputs at the end of the period as if they were outputs in that period. Figure 1 illustrates our formulation of the technology. Variable inputs x_t and quasi-fixed inputs k_{t-1} at the beginning of the period t are transformed by the process P_t into regular outputs y_t and quasi-fixed inputs k_t at the end of the period t. This implies that a firm cannot hold more quasi-fixed inputs without giving up a certain amount of products. In other words, a firm is subject to installation costs when it invests in quasi-fixed inputs. The more resources consumed in installing quasi-fixed inputs, the less there are left over for producing outputs.[2]

It should be noted that maintaining more quasi-fixed inputs transfers the current production to future periods because an increase in quasi-fixed inputs at the beginning of the period

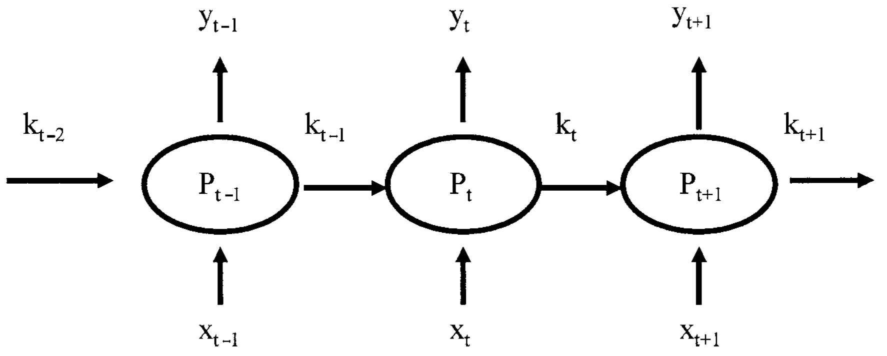


Figure 1. Technology of dynamic DEA.

raises production in that period. Given the technology with intertemporal substitution, our augmented DEA model determines the optimal allocation of production over time by minimizing the dynamic costs of a firm. As understood by Figure 1, our formulation of technology essentially corresponds to the basic dynamic technology proposed by Färe and Grosskopf (1996, Section 6.3).[3] Therefore, our dynamic DEA can be seen as a combination of the behavioral model of cost minimization and Färe and Grosskopf's basic dynamic technology.

The present paper is organized as follows. The augmented DEA model is specified as the primal LP problem in Section 1. Section 1 also presents a decomposition scheme of overall efficiency into static and dynamic components. In Section 2, applying the LP duality theorem and complimentary slackness conditions, we obtain the optimal condition for the adjustment path of quasi-fixed inputs. This optimality condition, which is the Hamilton-Jacobi-Bellman equation in terms of dynamic programming, provides a reference for measuring efficiencies in using variable and quasi-fixed inputs and in changing levels of quasi-fixed inputs. An empirical analysis is conducted in Section 3 using data pertaining to Japanese electric utilities over the 1981–1995 period. Finally, Section 4 provides a summary and conclusion.

2. Dynamic DEA Model

Let x_t denote a $l \times 1$ vector of variable inputs used in the period t, k_t a $m \times 1$ vector of quasi-fixed inputs at the end of the period t, and y_t a $n \times 1$ vector of outputs produced in the period t. A firm puts x_t and k_{t-1} into both production processes and investment activities so as to supply y_t to the market and to hold k_t at the end of the period. All combinations of $(x_t, k_{t-1}) \in \mathbb{R}_+^{l+m}$ and $(k_t, y_t) \in \mathbb{R}_+^{m+n}$, where the latter is transformable from the former, constitute the production possibility set in the period t:

$$\mathbf{\Phi}_t = \{(x_t, k_{t-1}, k_t, y_t) \in \mathbb{R}_+^{l+m} \times \mathbb{R}_+^{m+n} \mid (x_t, k_{t-1}) \text{ can yield } (k_t, y_t)\}. \tag{1}$$

It is required that $\mathbf{\Phi}_t$ satisfies the regularity conditions:

(i) if $(\tilde{x}_t, \tilde{k}_{t-1}, k_t, y_t) \in \mathbf{\Phi}_t$ and $(\tilde{x}_t, \tilde{k}_{t-1}) \leq (x_t, k_{t-1})$, then $(x_t, k_{t-1}, k_t, y_t) \in \mathbf{\Phi}_t$;

(ii) if $(x_t, k_{t-1}, \tilde{k}_t, \tilde{y}_t) \in \mathbf{\Phi}_t$ and $(\tilde{k}_t, \tilde{y}_t) \geq (k_t, y_t)$, then $(x_t, k_{t-1}, k_t, y_t) \in \mathbf{\Phi}_t$;

(iii) $\mathbf{\Phi}_t$ is closed and convex.

If the production technology is constant returns to scale, $\mathbf{\Phi}_t$ becomes a cone. That is,

(iv) if $(x_t, k_{t-1}, k_t, y_t) \in \mathbf{\Phi}_t$, then $(cx_t, ck_{t-1}, ck_t, cy_t) \in \mathbf{\Phi}_t$ for any $c > 0$.

Suppose that there is perfect foresight with respect to the input prices and the demands for products. Then, the intertemporal efficient frontier of costs follows as:

$$C(\bar{k}_0) = \min_{\{x_t,k_t\}_{t=1}^T} \left\{ \sum_{t=1}^{T} \gamma^t (w_t' x_t + v_t' k_{t-1}) \mid (x_t, k_{t-1}, k_t, y_t)_{t=1}^T \in \times_{t=1}^T \mathbf{\Phi}_t, k_0 = \bar{k}_0 \right\}, \tag{2}$$

where γ is a constant discount factor, w_t and v_t are $l \times 1$ and $m \times 1$ price vectors of variable and quasi-fixed inputs in the period t, respectively. Note that a bar "$^-$" indicates observed values exogenously given. The initial values of quasi-fixed inputs k_0 are given at $\bar{k}_0$, and the terminal values follow the natural boundary condition, i.e., T is fixed but k_T is free.[4]

To make (2) empirically amenable, DEA nonparametrically constructs a polyhedral convex set that approximates $\mathbf{\Phi}_t$ by enveloping observed data. Suppose that in the period t, there exist N observations regarding inputs and outputs: variable inputs, $X_t = (x_{t1}, x_{t2}, \ldots, x_{tN})$, quasi-fixed inputs at the beginning of the period t, $K_{t-1} = (k_{t-11}, k_{t-12}, \ldots, k_{t-1N})$, and quasi-fixed inputs at the end of the period t, $K_t = (k_{t1}, k_{t2}, \ldots, k_{tN})$. It is known that the smallest set including N observations and satisfying (i)–(iii) takes the form:

$$\hat{\mathbf{\Phi}}_t = \left\{ (x_t, k_{t-1}, k_t, y_t) \in \mathbb{R}_+^{l+m} \times \mathbb{R}_+^{m+n} \;\middle|\; X_t\lambda_t \le x_t,\ K_{t-1}\lambda_t \le k_{t-1}, \right.$$
$$\left. K_t\lambda_t \ge k_t,\ Y_t\lambda_t \ge y_t,\ \sum_{j=1}^{N} \lambda_{tj} = 1,\ \lambda_t \ge 0 \right\}, \tag{3}$$

where λ_t is a $N \times 1$ intensity vector whose jth element is denoted by λ_{tj}.[5] In a case of constant returns to scale technology, the smallest set including N observations and satisfying (i)–(iv) is obtained by removing the restriction $\sum_j \lambda_{tj} = 1$ from $\hat{\mathbf{\Phi}}_t$ defined above.

Replacing $\mathbf{\Phi}_t$ in (2) with $\hat{\mathbf{\Phi}}_t$, we calculate the intertemporal efficient frontier of costs from observed data. Specifically, the following LP problem is solved to yield an estimate of $C(\bar{k}_0)$:

$$\hat{C}(\bar{k}_0) = \min_{\{x_t,k_t,\lambda_t\}_{t=1}^T} \sum_{t=1}^{T} \gamma^t (w_t' x_t + v_t' k_{t-1}) \tag{4}$$

$$\begin{aligned}
\text{s.t.}\quad & X_t\lambda_t \le x_t, && t = 1, 2, \ldots, T \\
& K_{t-1}\lambda_t \le k_{t-1}, && t = 1, 2, \ldots, T \\
& K_t\lambda_t \ge k_t, && t = 1, 2, \ldots, T-1 \\
& Y_t\lambda_t \ge y_t, && t = 1, 2, \ldots, T \\
& i'\lambda_t = 1, && t = 1, 2, \ldots, T \\
& k_0 = \bar{k}_0, \quad x_t \ge 0, \quad k_t \ge 0, \quad \lambda_t \ge 0, && t = 1, 2, \ldots, T,
\end{aligned}$$

where i is a $N \times 1$ vector of ones.

The overall efficiency OE is measured as the ratio of $\hat{C}(\bar{k}_0)$ to the corresponding actual costs. That is,

$$\mathrm{OE} = \hat{C}(\bar{k}_0)/C, \tag{5}$$

where C is the discounted sum of actual costs over the period from 1 to T. Here, what we mean by "overall" is twofold. First, OE is overall because it reflects accumulated inefficiency over the period from 1 to T. Taking the terminal period T as variable, we will show in the next section how OE evolves for $t = 1, 2, \ldots, T$. Second, OE can be factorized in a parallel way to decomposing the cost efficiency into technical and allocative efficiencies in the static frontier models. In the rest of this section, we provide an extended scheme in which the overall efficiency is decomposed into static technical efficiency TE, static allocative efficiency AE, and dynamic efficiency DE. TE and AE are static measures because they embody inefficiencies originating from the levels of variable inputs, while DE embodies inefficiency originating from the paths of quasi-fixed inputs.

The decomposition proceeds as follows. First of all, the static efficiency is isolated from the overall efficiency by holding quasi-fixed inputs at observed levels in (4). The dynamic efficiency DE is then defined as a residual left after removing the static efficiency from OE. On the other hand, the static efficiency is further decomposed into technical efficiency TE and allocative efficiency AE. Like the static DEA, TE is defined as the level of possible reduction in costs resulting from a uniform radial contraction of all variable inputs. Finally, AE is calculated as a residual left after removing TE from the static efficiency.

More formally, static efficiency is represented by the efficient costs given quasi-fixed inputs at observed levels:

$$C_{SE} = \min_{\{x_t\}_{t=1}^T} \left\{ \sum_{t=1}^{T} \gamma^t (w_t' x_t + v_t' \bar{k}_{t-1}) \,\middle|\, (x_t, k_{t-1}, k_t, y_t)_{t=1}^T \in \times_{t=1}^T \boldsymbol{\Phi}_t \right\}. \tag{6}$$

The difference between C_{SE} and the actual costs C is due to the inefficient use of variable inputs because quasi-fixed inputs are held at observed levels for C_{SE}. The measure of static efficiency SE is thus defined by

$$\mathrm{SE} = C_{SE}/C. \tag{7}$$

Similarly, the difference between C_{SE} and the fully efficient costs $C(\bar{k}_0)$ is due to an inefficient choice of the path of quasi-fixed inputs. The measure of dynamic efficiency is thus defined by

$$\mathrm{DE} = C(\bar{k}_0)/C_{SE}. \tag{8}$$

In practice, $C(\bar{k}_0)$ and C_{SE} are replaced with $\hat{C}(\bar{k}_0)$ and $\hat{C}_{SE}$, respectively, and $\hat{C}_{SE}$ is obtained by solving the analytic LP problem:

$$\hat{C}_{SE} = \min_{\{x_t, \lambda_t\}_{t=1}^T} \sum_{t=1}^{T} \gamma^t (w_t' x_t + v_t' \bar{k}_{t-1}) \tag{9}$$

$$\begin{aligned} \text{s.t.}\ \ & X_t \lambda_t \leq x_t, && t = 1, 2, \ldots, T \\ & K_{t-1} \lambda_t \leq \bar{k}_{t-1}, && t = 1, 2, \ldots, T \\ & K_t \lambda_t \geq \bar{k}_t, && t = 1, 2, \ldots, T-1 \\ & Y_t \lambda_t \geq y_t, && t = 1, 2, \ldots, T \\ & i' \lambda_t = 1, && t = 1, 2, \ldots, T \\ & x_t \geq 0, \lambda_t \geq 0, && t = 1, 2, \ldots, T. \end{aligned}$$

It should be noted that DE includes forecasting errors for input prices and demands for outputs in the future. Unless the firm's forecasts on future variables are substituted into (4), $C(\bar{k}_0)$ would be less complete as a behavioral model.[6] However, the primary purpose here is not to represent the real behavior of a firm by $C(\bar{k}_0)$ but to represent the best practice as a reference to which the dynamic efficiency is evaluated.

Next, SE is further decomposed into TE and AE. TE is formally defined by the variable input distance function:

$$D_t(x_t, y_t; \bar{k}_t, \bar{k}_{t-1}) = \max\{\zeta \mid (x_t/\zeta, \bar{k}_t, \bar{k}_{t-1}) \in \mathbf{\Phi}_t\}. \tag{10}$$

Letting, $\phi_t = D_t^{-1}$, we can write the ray minimum costs with fixed quasi-fixed inputs as:

$$C_{TE} = \sum_{t=1}^{T} \gamma^t (w_t' \bar{x}_t \phi_t + v_t' \bar{k}_{t-1}), \tag{11}$$

The static technical efficiency is then measured by

$$\text{TE} = C_{TE}/C. \tag{12}$$

In practice, C_{TE} is replaced with $\hat{C}_{TE}$ obtained by solving the following analytic LP problem:

$$\hat{C}_{TE} = \min_{\{\phi_t, \lambda_t\}_{t=1}^T} \sum_{t=1}^{T} \gamma^t (\phi_t w_t' \bar{x}_t + v_t' \bar{k}_{t-1}) \tag{13}$$

$$\begin{aligned} \text{s.t.}\ \ & X_t \lambda_t \leq \phi_t \bar{x}_t, && t = 1, 2, \ldots, T \\ & K_{t-1} \lambda_t \leq \bar{k}_{t-1}, && t = 1, 2, \ldots, T \\ & K_t \lambda_t \geq \bar{k}_t, && t = 1, 2, \ldots, T-1 \\ & Y_t \lambda_t \geq y_t, && t = 1, 2, \ldots, T \\ & i' \lambda_t = 1, && t = 1, 2, \ldots, T \\ & \phi_t \geq 0, \lambda_t \geq 0, && t = 1, 2, \ldots, T. \end{aligned}$$

Here, the radial contraction rate ϕ_t is allowed to vary over the periods. Since quasi-fixed inputs are exogenously given at the actual levels, there are no restrictions across the periods. Thus, the LP problem (13) is reduced to T single period problems that are independent of each other.

As commonly conducted in the frontier models, allocative efficiency is isolated by removing technical efficiency from cost efficiency. We define static allocative efficiency AE as

$$\mathrm{AE} = C_{SE}/C_{TE}. \tag{14}$$

AE reflects the costs that could be saved if variable inputs were adjusted to the optimal levels along the short-run isoquant.

From eqs. (5), (7), (8), (12) and (14), we eventually have a multiplicative relationship:

$$\mathrm{OE} = \mathrm{TE} \cdot \mathrm{AE} \cdot \mathrm{DE}. \tag{15}$$

In Section 3, the decomposition analysis based on (15) will be illustrated with an application to Japanese electric utilities.

3. Optimality Condition

Following Nemoto and Goto (1999), this section derives the results related to the conditions of dynamic optimality as far as required in Section 3. For this purpose, consider the dual problem to (4):

$$J_T(\bar{k}_0) = \max_{\{\alpha_t,\beta_t,\mu_t,\theta_t,\epsilon_t\}_{t=1}^T} \gamma v_1' \bar{k}_0 - \beta_1' \bar{k}_0 + \sum_{t=1}^T \mu_t' y_t + \sum_{t=1}^T \epsilon_t \tag{16}$$

$$\begin{aligned}
\text{s.t. } & \alpha_t' \leq \gamma^t w_t', && t = 1, 2, \ldots, T \\
& -\alpha_t' X_t - \beta_t' K_{t-1} + \theta_t' K_t + \mu_t' Y_t + i'\epsilon_t \leq 0, && t = 1, 2, \ldots, T \\
& \beta_t' - \theta_{t-1}' \leq \gamma^t v_t', && t = 2, 3, \ldots, T \\
& \alpha_t \geq 0, \beta_t \geq 0, \mu_t \geq 0, && t = 1, 2, \ldots, T \\
& \theta_t \geq 0, \quad t = 1, 2, \ldots, T-1, \quad \theta_T = 0.
\end{aligned}$$

Note that ϵ_t is an unrestricted scalar in sign because the corresponding constraint is an equality $i'\lambda_t = 1$. Letting an asterisk indicate an optimal solution for the primal and dual problems, (4) and (16), it follows from the complementary slackness conditions that:

$$(\gamma^t w_t - \alpha_t^*)' x_t^* = 0, \qquad t = 1, 2, \ldots, T; \tag{17}$$

$$(\alpha_t^{*\prime} X_t + \beta_t^{*\prime} K_{t-1} - \theta_t^{*\prime} K_t - \mu_t^{*\prime} Y_t - i'\epsilon_t^*)\lambda_t^* = 0, \qquad t = 1, 2, \ldots, T; \tag{18}$$

$$(\gamma^t v_t - \beta_t^* + \theta_{t-1}^*)' k_{t-1}^* = 0, \qquad t = 2, 3, \ldots, T; \tag{19}$$

$$\alpha_t^{*\prime}(X_t \lambda_t^* - x_t^*) = 0, \qquad t = 1, 2, \ldots, T; \tag{20}$$

$$\beta_t^{*\prime}(K_{t-1}\lambda_t^* - k_{t-1}^*) = 0, \qquad t = 1, 2, \ldots, T; \tag{21}$$

$$\mu_t^{*\prime}(y_t - Y_t \lambda_t^*) = 0, \qquad t = 1, 2, \ldots, T; \tag{22}$$

$$\theta_t^{*\prime}(k_t^* - K_t \lambda_t^*) = 0, \qquad t = 1, 2, \ldots, T, \tag{23}$$

where $k_0^* = \bar{k}_0$ and $\theta_T^* = 0$. Substituting (20)–(23) into (18) yields

$$\alpha_t^{*\prime} x_t^* + \beta_t^{*\prime} k_{t-1}^* - \theta_t^{*\prime} k_t^* - \mu_t^{*\prime} y_t = i'\epsilon_t^* \lambda_t^*, \quad t = 1, 2, \ldots, T. \tag{24}$$

Using (17) and (19) and recalling $i'\lambda_t^* = 1$, we further rewrite (24) as

$$\gamma^t w_t' x_t^* + \gamma^t v_t' k_{t-1}^* - \theta_t^{*\prime} k_t^* + \theta_{t-1}^{*\prime} k_{t-1}^* - \mu_t^{*\prime} y_t = \epsilon_t^*, \quad t = 1, 2, \ldots, T. \tag{25}$$

Here, θ_0^* is defined to be $\beta_1^* - \gamma v_1$. This definition is natural because $\theta_{t-1}^* = \beta_t^* - \gamma^t v_t$ holds for $t \geq 2$ from (19) if $K_{t-1} > 0$ for $t \geq 2$ and thereby $k_{t-1}^* > 0$ for $t \geq 2$.

Equation (25) describes the path along which the optimal values of variable and quasi-fixed inputs evolve. In fact, equation (25) can be seen as the Hamilton-Jacobi-Bellman equation in terms of the dynamic programming. This is clarified in the appendix. Equation (25) is also compatible with the adjustment-cost theory of investment. Nemoto and Goto (1999) show that the marginal adjustment costs are retrieved by θ_t^* as well as the marginal products of quasi-fixed inputs by $\beta_t^* - (1 - \delta)\theta_t^*$ where δ is a deterioration rate.[7]

Those theoretically favorable features lead us to employ equation (25) as a reference for measuring the inefficiencies in using variable and quasi-fixed inputs and in changing the levels of quasi-fixed inputs. We thus define the inefficiency measures in the period t for variable inputs τ_t^x, for quasi-fixed inputs τ_t^k, and for net investment in quasi-fixed inputs τ_t^h as:

$$\begin{aligned} \tau_t^x &= \gamma^t w_t'(x_t - x_t^*)/C, & t &= 1, 2, \ldots, T; \\ \tau_t^k &= \gamma^t v_t'(k_{t-1} - k_{t-1}^*)/C, & t &= 2, 3, \ldots, T; \\ \tau_t^h &= \{(\theta_t^{*\prime} k_t - \theta_{t-1}^{*\prime} k_{t-1}) - (\theta_t^{*\prime} k_t^* - \theta_{t-1}^{*\prime} k_{t-1}^*)\}/C, & t &= 2, 3, \ldots, T-1, \end{aligned} \tag{26}$$

where x_t, k_t and k_{t-1} are evaluated at observed values, and C is the discounted sum of actual total costs over the planning period. By construction, positive (negative) values of those measures indicate excess (short) usage of inputs or excess (short) net investment. Evidently, equation (26) measures the inefficiencies according to the normalized deviations of observations along (25) from the optimal input usage and net investment. Moreover, equation (26) may be viewed as formulas aggregating the inefficiencies of individual inputs and net investments with weights $\gamma^t w_t$, $\gamma^t v_t$, θ_t^* and θ_{t-1}^*.

The aggregate inefficiency measures are then normalized by C so that they are linked to the overall efficiency, OE, defined in the last section. Summing $\tau_t^x + \tau_t^k + \tau_t^h$ over time yields

$$\sum_{t=1}^{T} \left(\tau_t^x + \tau_t^k + \tau_t^h\right) = 1 - \mathrm{OE} - \frac{1}{C}\{\theta_T^{*\prime}(k_T - k_T^*) - \theta_0^{*\prime}(\bar{k}_0 - k_0^*)\}. \tag{27}$$

The first term in the braces is zero as $\theta_T^* = 0$ by the terminal condition. The second term in the braces also becomes zero as $\bar{k}_0 = k_0^*$ by the initial condition. Therefore, we have

$$1 - \sum_{t=1}^{T} \left(\tau_t^x + \tau_t^k + \tau_t^h\right) = \mathrm{OE}, \tag{28}$$

which provides another decomposition of OE. It should be noted that the sum of τ_t^h over the whole period is equal to zero. This is not surprising because any inefficiencies due to the levels of quasi-fixed inputs are drawn by τ_t^k. The inefficiencies evaluated by τ_t^h entirely concern the allocation of net investment during the planning period while the total amounts of net investment are given. The levels of quasi-fixed inputs determined by the accumulation of net investment are evaluated by τ_t^k.

In addition, equation (28) shows the time development of the overall efficiency. Taking the terminal period T as variable, we can rewrite (28) as

$$\mathrm{OE}_t = \mathrm{OE}_{t-1} - \left(\tau_t^x + \tau_t^k + \tau_t^h\right), \tag{29}$$

where OE_t is the overall efficiency over the period from 1 to t. The overall efficiency is nonincreasing in time because it depreciates every period by the sum of inefficiencies that occurred in that period.

4. Empirical Implementation

In this section, we illustrate the empirical usefulness of the proposed dynamic DEA with an application to Japanese electric utilities.

4.1. Data

The data set consists of a total of 135 observations of nine privately-owned Japanese electric utilities during the 1981–1995 period. Over this period, Japanese electric utilities were vertically integrated and locally monopolized under the rate-of-return regulation. An assumption of cost minimization is thus considered to be plausible because the demands in a franchise area are given at the regulated prices.

Electric utility firms are supposed to provide two outputs with two variable inputs and three quasi-fixed inputs. One of the two outputs is electricity for commercial and industrial use, and the other is electricity for residential use. They are measured by the amounts of electricity in megawatt-hours (MWh) sold to respective customers.

The two variable inputs include fuel and labor. Fuel input is measured by the total kilocalorie content of coal, natural gas, petroleum and nuclear fuels. The price of fuel is fuel expenses divided by fuel input. Labor input is the number of full-time employees plus the adjusted number of part-time employees, where the latter is calculated by (the number of part-time employees) $\times$ (wages and salaries paid for part-time employees)/(wages and salaries paid for full-time employees). The price of labor is total wages and salaries divided by the labor input.

The three quasi-fixed inputs include generation plants, transmission facilities and distribution facilities. We measure them by physical units: generation plants are measured by the total nameplate capacity in megawatts (MW); transmission facilities by the weighted sum of the circuit length of transmission lines (km) with their mid-point voltage (kV) as weights; and distribution facilities by the total transformation capacity for distribution (MVA). The service prices of quasi-fixed inputs for $t \geq 2$ are constructed by the following formula:

$$v_{ti} = u_{ti}\{r + \delta_{ti} - (1+r)(u_{ti} - u_{t-1i})/u_{ti}\}, \quad t = 2, 3, \ldots, T,$$

where v_{ti}, u_{ti} and δ_{ti} are, respectively, service price, acquisition price and deterioration rate of the ith quasi-fixed inputs, and r is the nominal discount rate that relates the discount factor as $\gamma = 1/(1+r)$.[8] This formula ensures that the objective of (4), $\sum \gamma^t (w_t' x_t + v_t' k_{t-1})$, is approximately equal to the discounted sum of expenses for variable inputs and gross investment in quasi-fixed inputs.[9] As easily verified, the service prices conform to Jorgenson's capital user's cost. In fact, a well-known form of $u_{ti}(r + \delta_{ti})$ is recovered if acquisition prices are constant over time. The acquisition prices are calculated by investment expenses per net investment in physical units. The nominal discount rate is assumed to be constant at 0.06.

All data used in this paper are drawn from the annual financial statements of the nine Japanese electric utilities and relevant issues of the *Handbook of the Electric Power Industry* published annually by the Federation of Electric Power Companies of Japan.

4.2. Results

Using equation (15) from Section 1, we first decompose the overall efficiency of the Japanese electric utilities. To construct an empirical production possibility set, a two-year window is applied to the reference sets for inputs and outputs. That is, the empirical production possibility set employed in this paper is spanned by the vectors of inputs and outputs observed in the current and last years. Specifically,

$$\begin{aligned} X_t &= (x_{t-11}, x_{t-12}, \ldots, x_{t-19}, x_{t1}, x_{t2}, \ldots, x_{t9}); \\ K_t &= (k_{t-11}, k_{t-12}, \ldots, k_{t-19}, k_{t1}, k_{t2}, \ldots, k_{t9}); \\ Y_t &= (y_{t-11}, y_{t-12}, \ldots, y_{t-19}, y_{t1}, y_{t2}, \ldots, y_{t9}). \end{aligned}$$

Here, x_{ti} and y_{ti} are, respectively, the observed variable input and output vectors for the ith electric utility firm in the period t; k_{ti} is the observed quasi-fixed input vector for the ith firm at the end of the period t. We impose additional restrictions on μ in order to restrict the shadow prices of outputs within the observed range of their unit values.[10] The planning period covers from 1981 to 1995. Thus, $\bar{k}_0$ corresponds to the initial stock at the beginning of 1981 and k_T the terminal stock at the end of 1995. The discount factor is set at 0.9434 so that the nominal discount rate is 0.06.

Table 1 shows the results of decomposition based on the dynamic DEA, together with conventional decomposition based on the static DEA for comparison. In Table 1, efficiency scores are all calculated using the production possibility set of constant returns to scale technology, i.e., $i'\lambda_t = 1$ is removed from the set of constraints in (3). From the second to fifth columns, scores of overall efficiency and its decomposition are displayed. The OE scores range from 0.765 to 0.998, indicating that, without any inefficiencies, total cost reductions over the planning period of 0.2–23.5 percent would be possible in terms of the present value. The decomposition of OE shows that most of TE and AE are unity, and thereby DE is very close to OE. This implies that little inefficiency is attributable to variable inputs, and that the fixity of quasi-fixed inputs is almost the only source of overall inefficiency. As a result, static DEA may be considered to give biased results because it ignores the fixity of quasi-fixed inputs.

Table 1. Decomposition of overall efficiency (constant returns to scale).

	Dynamic DEA				Static DEA		
Company	OE	TE	AE	DE	OE^S	TE^S	AE^S
Hokkaido	0.861	1.000	1.000	0.862	0.844	0.975	0.865
Tohoku	0.836	1.000	0.997	0.838	0.807	0.981	0.822
Tokyo	0.998	1.000	1.000	0.998	0.988	0.998	0.989
Chubu	0.928	1.000	1.000	0.928	0.912	0.998	0.914
Hokuriku	0.880	0.999	0.999	0.882	0.810	0.990	0.818
Kansai	0.843	1.000	1.000	0.843	0.831	0.998	0.833
Chugoku	0.765	0.991	0.996	0.775	0.731	0.838	0.872
Shikoku	0.965	1.000	1.000	0.965	0.838	0.995	0.842
Kyushu	0.796	0.995	0.999	0.800	0.757	0.890	0.851

Note:
OE = TE ∗ AE ∗ DE (dynamic DEA).
OE: overall efficiency.
TE: technical efficiency.
AE: allocative efficiency.
DE: dynamic efficiency.
$OE^S = TE^S * AE^S$ (static DEA).
OE^S: overall efficiency.
TE^S: technical efficiency.
AE^S: allocative efficiency.

To confirm this, the efficiency scores are computed with static DEA. Treating all inputs as variable, we here define static DEA-based efficiency measures as follows:

$$\begin{aligned}
\mathrm{OE}^S &= \min_{x_t,k_t,\lambda_t} \left\{ \sum_{t=1}^{T} \gamma^t (w_t' x_t + v_t' k_{t-1}) \middle/ \bar{C} \,\middle|\, \begin{pmatrix} X_t \\ K_{t-1} \end{pmatrix} \lambda_t \le \begin{pmatrix} x_t \\ k_{t-1} \end{pmatrix}, \right. \\
&\qquad \left. Y_t \lambda_t \ge y_t,\ t = 1, 2, \ldots, T \right\}; \\
\mathrm{TE}^S &= \min_{\phi_t,\lambda_t} \left\{ \sum_{t=1}^{T} \gamma^t \phi_t (w_t' \bar{x}_t + v_t' \bar{k}_{t-1}) \middle/ \bar{C} \,\middle|\, \begin{pmatrix} X_t \\ K_{t-1} \end{pmatrix} \lambda_t \le \phi_t \begin{pmatrix} \bar{x}_t \\ \bar{k}_{t-1} \end{pmatrix}, \right. \\
&\qquad \left. Y_t \lambda_t \ge y_t,\ t = 1, 2, \ldots, T \right\}; \\
\mathrm{AE}^S &= \mathrm{OE}^S / \mathrm{TE}^S,
\end{aligned}$$

where TE^S, AE^S and OE^S are the technical, allocative and overall efficiencies, respectively. It can be seen that these measures follow the conventional definitions of cost efficiency except that the above measures evaluate multiperiod costs. The scores of OE^S, TE^S and AE^S are displayed in Table 1 from the sixth to the eighth columns.

In comparison, the degree of overall efficiency implied by OE^S is quite similar to that of its dynamic counterpart measured by OE in seven of the nine firms. The two exceptions are Hokuriku and Shikoku: their OE^S scores understate the overall efficiency by 0.07–0.13

points. In agreement with the dynamic DEA-based TE, TE^S scores (except those from Chugoku and Kyushu) show that Japanese electric utilities are technically efficient. In contrast, AE^S scores excluding Tokyo range from 0.82 to 0.91, which are rather lower than the dynamic DEA-based AE. As a result, static DEA indicates that the most important contributor to the overall inefficiency of Japanese electric utilities is an allocative one, with technical inefficiency being equally important for Chugoku and Kyushu. Unfortunately, these results are misleading. The dynamic DEA indicates that overall inefficiency originates solely from dynamic inefficiency, i.e., an inadequate intertemporal allocation of quasi-fixed inputs. Evidently, static DEA incorrectly attributes the source of overall inefficiency to static allocative inefficiency by ignoring the short-run fixity of quasi-fixed inputs.[11] As a result, static DEA will mislead the regulatory authority and electric utility firms into taking hasty steps to adjust all inputs. However, this is likely to cause an excessive adjustment of quasi-fixed inputs over the optimal one indicated by dynamic DEA. Our results also suggest that the regulatory authority must pay close attention to an incentive scheme for investment.

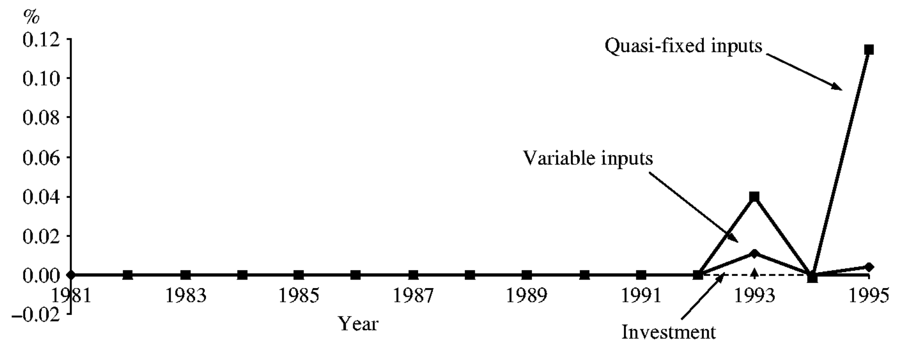
Next, Table 2 reports the results of measuring efficiency scores subject to variable returns to scale technology. All constraints in (3) are utilized to construct the production possibility set. Scores in Table 2 are higher than in Table 1 because the production possibility set becomes smaller with variable returns to scale. The fact that dynamic inefficiency alone contributes to overall inefficiency, however, is not altered. Furthermore, switching the production technology does not affect the rank of OE except to drop Chubu from third to fifth place. We, thus, proceed with the constant returns to scale in the following analysis.

Figures 2(a–c) present the development over time of inefficiency measured by deviations from the optimality condition, τ_t^x, τ_t^k and τ_t^h in (26), for three selected firms that exhibit typical patterns. The results for Tokyo are shown in Figure 2(a). The aggregate measures

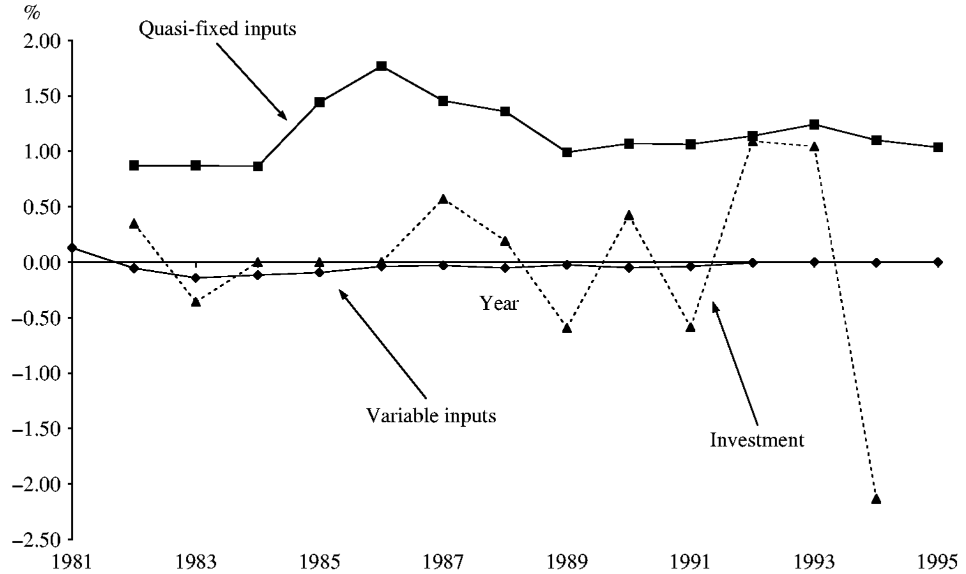
Table 2. Decomposition of overall efficiency (variable returns to scale).

	Dynamic DEA			
Company	OE	TE	AE	DE
Hokkaido	0.966	1.000	1.000	0.966
Tohoku	0.886	1.000	1.000	0.886
Tokyo	1.000	1.000	1.000	1.000
Chubu	0.949	1.000	1.000	0.949
Hokuriku	0.989	1.000	1.000	0.989
Kansai	0.859	1.000	1.000	0.859
Chugoku	0.821	0.995	0.999	0.826
Shikoku	1.000	1.000	1.000	1.000
Kyushu	0.834	0.998	0.999	0.837

Note:
OE = TE ∗ AE ∗ DE.
OE: overall efficiency.
TE: technical efficiency.
AE: allocative efficiency.
DE: dynamic efficiency.



(a)

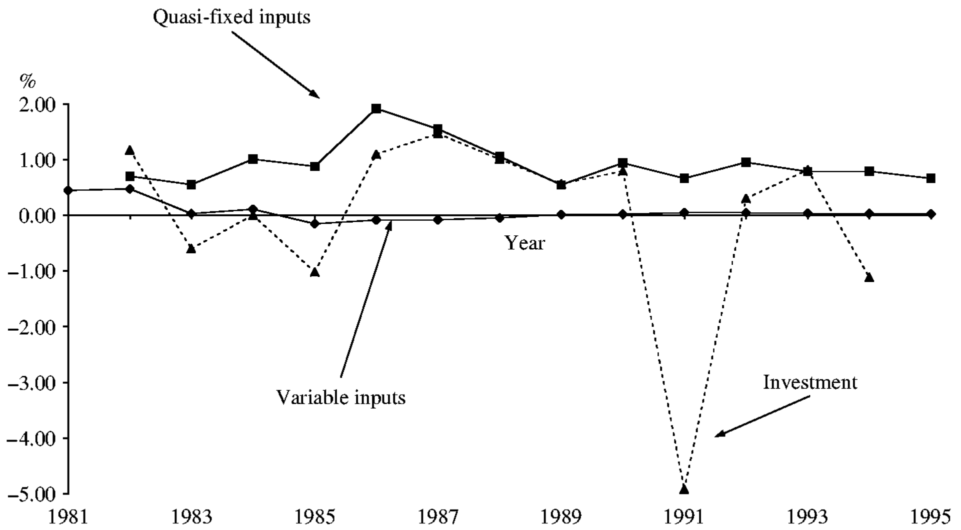


(b)

Figure 2. (a) Inefficiency in variable inputs (τ_t^x), quasi-fixed inputs (τ_t^k) and investment (τ_t^h) measured by deviations from the optimal state for Tokyo company. (b) Inefficiency in variable inputs (τ_t^x), quasi-fixed inputs (τ_t^k) and investment (τ_t^h) measured by deviations from the optimal state for Kansai company. (c) Inefficiency in variable inputs (τ_t^x), quasi-fixed inputs (τ_t^k) and investment (τ_t^h) measured by deviations from the optimal state for Hokkaido company.

of variable inputs, τ_t^x, quasi-fixed inputs, τ_t^k, and net investment in quasi-fixed inputs, τ_t^h, all designate very slight deviations from the optimal state. This is consistent with the rating of OE for Tokyo reported in Table 1.

The results for Kansai are shown in Figure 2(b). Similar to Tokyo, variable inputs have been close to the optimal levels as τ_t^x is nearly zero. In contrast, quasi-fixed inputs are



(c)

Figure 2. (*Continued*)

found to be persistently excessive. The aggregate measure τ_t^k indicates an excess holding of quasi-fixed inputs to the extent of 0.9–1.8 percent of the present value of total costs over the planning period. On the other hand, the aggregate measure of net investment τ_t^h oscillates around zero after the late 1980s. This may support the usual econometric specifications of the optimal equation with a symmetric error term, though heteroscedasticity seems to exist.

Figure 2(c) presents the results for Hokkaido. The time pattern of τ_t^h greatly differs from that in both Tokyo and Kansai: there is a sharp drop in 1991 that compensates overinvestment in the other years. Such a development of τ_t^h seems to suggest a discrete adjustment due to the indivisibility of generation, transmission and distribution facilities. The development of τ_t^x and τ_t^k in Figure 2(c) corresponds to that of Kansai in Figure 2(b), designating an efficient usage of variable inputs and excess holding of quasi-fixed inputs.

The excess holding of capital stocks or overcapitalization in a regulated industry has received much attention in the literature. To see this more precisely, we further decompose the aggregate measure τ_t^k into each component:

$$\tau_t^{ki} = \gamma^t v_{ti}'(k_{t-1i} - k_{t-1i}^*)/C, \quad i = 1, 2, 3,$$

where generation, transmission, and distribution sectors are denoted by $i = 1, 2,$ and 3, respectively.

Figures 3(a–c) present the development of τ_t^{ki}, $i = 1, 2, 3$ for the Tokyo, Kansai and Hokkaido companies. Figure 3(a) shows that Tokyo has not overcapitalized in any sectors over time except for some slight deviations from optimality in 1993 and 1995. This is

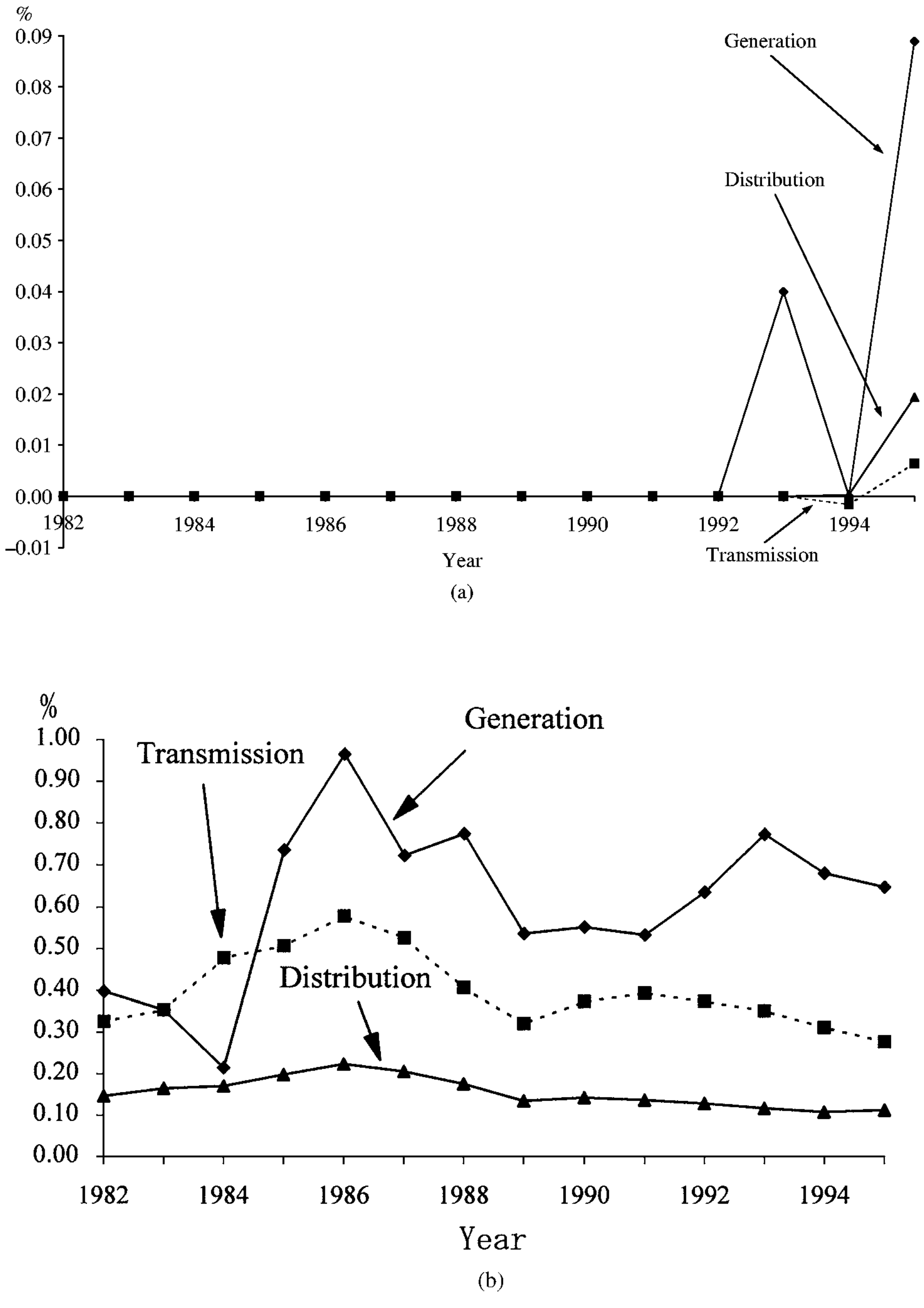
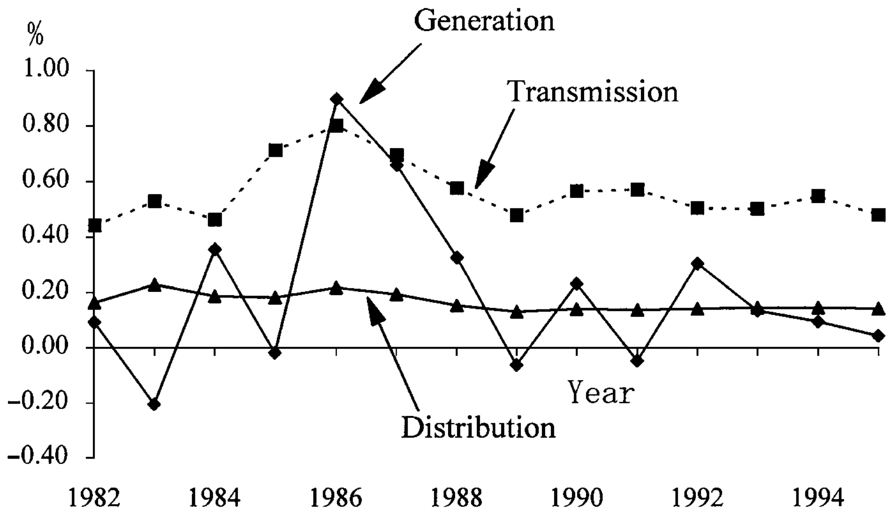


Figure 3. (a) Deviations of quasi-fixed inputs from their optimal levels in Tokyo company. (b) Deviations of quasi-fixed inputs from their optimal levels in Kansai company. (c) Deviations of quasi-fixed inputs from their optimal levels in Hokkaido company.



(c)

Figure 3. (*Continued*)

consistent with the rating of DE for Tokyo shown in Table 1. It can be seen from Figures 3(b) and 3(c) that, for Kansai and Hokkaido, the generation and transmission sectors maintain excess facilities while the distribution sectors hold relatively efficient facilities. The largest overcapitalization is found in the transmission sector for Hokkaido and in the generation sector for Kansai. This may be a reflection of the difference in the efficiency of the transmission network between the two companies due to the much lower demand density in Hokkaido.

5. Conclusion

This paper investigates the use of a nonparametric analysis of productive efficiency within a dynamic framework. We show how dynamic data envelopment analysis can decompose overall efficiency into static and dynamic efficiencies. We also show how the intertemporal optimality condition is derived and used as a reference to measure inefficiencies due to variable inputs, quasi-fixed inputs and changing levels of quasi-fixed inputs.

The proposed procedure is illustrated in an application to Japanese electric utilities. The results indicate that Japanese electric utilities are efficient in their use of variable inputs, and that this inefficiency is attributable to a failure in adjusting quasi-fixed inputs to their optimal levels. An excess holding of quasi-fixed inputs is also detected in several firms. These findings suggest that dynamic data envelopment analysis is a promising tool for analyzing the dynamic aspects of production.

Moreover, it is noteworthy that several extensions are possible. First, switching the assumption on a firm's behavior from cost minimization to profit maximization is straightforward. Profit maximizing behavior is formulated by the analytic LP problem in which the discounted sum of net cash flow is maximized under the same restrictions as equation (4). Unlike the cost minimizing model, outputs become endogenous variables chosen by a firm. Thus, the profit maximizing model can be consistent with an output-oriented approach to technical efficiency.

Second, regulatory effects on the use of inputs in public utility firms are modeled by adequately restricting the feasible domain of inputs and outputs. For the rate of return regulation, Färe and Logan (1992) show how the feasible set of inputs and outputs is adapted to the DEA framework. Their notion of a regulated input distance function is immediately applicable to our dynamic DEA. If the pricing behavior of a firm is incorporated, the price-cap regulation may also be handled by the dynamic DEA.

Last, an assumption that all investments instantaneously become productive can be removed by distinguishing productive quasi-fixed inputs from those under construction. Let $\tilde{k}_t$ denote all quasi-fixed inputs including construction in progress at the end of the period t. The relationship between $\tilde{k}_t$ and productive quasi-fixed inputs k_t depends on the pattern of time lag in the installation of quasi-fixed inputs. If some of the net investments becomes productive with a one-period lag and the rest becomes immediately productive, we have $k_t = \pi(\tilde{k}_t - \tilde{k}_{t-1}) + \tilde{k}_{t-1}$ if $\tilde{k}_t \geq \tilde{k}_{t-1}$, where π is a diagonal matrix with diagonal elements representing fractions of the net investments that become immediately productive.[12] Introducing this restriction into the analytic LP problem, we may handle the effects of time to build delays in investment in quasi-fixed inputs.[13]

Notwithstanding its usefulness and potential value, the shortcomings of our method should be kept in mind. One of its most important limitations is the assumption of perfect foresight for future variables. There may be at least two routes to circumvent this problem. First, to the extent that an assumption of perfect foresight is unrealistic, the resulting inefficiency scores should include forecasting errors. To evaluate pure inefficiencies, the components of forecasting errors may be removed. Using the orthogonal property of forecasting errors to an information set, we may isolate them and extract the pure inefficiencies. Second, the techniques of time series modelling may be helpful in estimating conditional expected values for future demands and inputs. Once these conditional expected values are obtained, replacing the corresponding future variables with them enables us to apply the certainty equivalence principle to solving the stochastic LP problem. Although stochastic DEA is beyond the scope of the present study, it deserves future research. The present analysis can be extended further to a stochastic DEA in dynamic production processes.

Appendix

This appendix serves to clarify the economic implications of optimal solutions of the analytic LP problems (4) and (16). We show that equation (25) is interpretable as the Hamilton-Jacobi-Bellman equation of the intertemporal cost minimization behavior of a firm. Furthermore, we will find that if a firm maximizes its value, β_1^* represents the marginal value of the initial quasi-fixed inputs and is closely related to Tobin's marginal q.

A. Hamilton-Jacobi-Bellman Equation

Note that the LP problem (4) can be reformulated in a recursive form as:

$$\hat{C}_{t-1}(k_{t-1}) = \min_{x_t, k_t}\{\gamma^t(w_t' x_t + v_t' k_{t-1}) + \hat{C}_t(k_t) \mid (x_t, k_{t-1}, k_t, y_t) \in \hat{\Phi}_t\}. \tag{A.1}$$

Then, the Hamilton-Jacobi-Bellman equation for (A.1) can be written as:

$$\gamma^t w_t' x_t^* + \gamma^t v_t' k_{t-1}^* + \hat{C}_t(k_t^*) - \hat{C}_{t-1}(k_{t-1}^*) = 0. \tag{A.2}$$

The LP duality theorem ensures that the minimand of (4) is identical to the maximand of (16) at the optimum. Thus, substituting $\hat{C}_{t-1}(k_{t-1}^*) = J_{t-1}(k_{t-1}^*)$ into (A.2), we have

$$\gamma^t w_t' x_t^* + \gamma^t v_t' k_{t-1}^* + J_t(k_t^*) - J_{t-1}(k_{t-1}^*) = 0. \tag{A.3}$$

Recall that by definition,

$$J_{t-1}(k_{t-1}^*) = \gamma^t v_t' k_{t-1}^* - \beta_t^{*\prime} k_{t-1}^* + \sum_{j=t}^{T} \mu_j^{*\prime} y_j + \sum_{j=t}^{T} \epsilon_j^*. \tag{A.4}$$

As stated in the main text, the third constraint in the LP problem (16) holds by equality at the optimum as:

$$\theta_{t-1}^* = \beta_t^* - \gamma^t v_t, \quad t = 2, 3, \ldots, T, \tag{A.5}$$

if K_{t-1} for $t \geq 2$ contains only positive elements. Substituting (A.5) into (A.4) and combining the resulting form with (A.3), we have

$$\gamma^t w_t' x_t^* + \gamma^t v_t' k_{t-1}^* - \theta_t^{*\prime} k_t^* + \theta_{t-1}^{*\prime} k_{t-1}^* - \mu_t^{*\prime} y_t = \epsilon_t^*,$$

which is equation (25). Thus, equation (25) is found to be equivalent to the Hamilton-Jacobi-Bellman equation characterizing the intertemporal optimization behavior of a firm.

B. Tobin's Marginal q

Next, let us define the value function of a firm by the discounted sum of net cash flow with its scrap value at the terminal period:

$$V(\bar{k}_0) \equiv \sum_{t=1}^{T} \gamma^t \{p_t' y_t - w_t' x_t - u_t'(k_t - k_{t-1} + D k_{t-1})\} + \gamma^T u_T' k_T, \tag{A.6}$$

where p_t denotes a $n \times 1$ vector of output prices, u_t a $m \times 1$ vector of acquisition prices of quasi-fixed inputs, D a $m \times m$ diagonal matrix with deterioration rates of quasi-fixed inputs on the diagonal and I a $m \times m$ identity matrix. We can rewrite (A.6) as:

$$V(\bar{k}_0) = \sum_{t=1}^{T} \gamma^t p_t' y_t - \sum_{t=1}^{T} \gamma^t w_t' x_t - \sum_{t=2}^{T} \gamma^t v_t' k_{t-1} - \gamma u_1'(D - I) k_0. \tag{A.7}$$

Evidently, the last three terms of this expression are identical to $\hat{C}_0(\bar{k}_0)$ if v_1 is defined to be $u_1'(D-I)$. Since $\hat{C}_0(\bar{k}_0) = J_0(\bar{k}_0)$ at the optimum, the value function finally takes the form:

$$
\begin{aligned}
V(\bar{k}_0) &= \sum_{t=1}^{T} \gamma^t p_t' y_t - J_0(\bar{k}_0) \\
&= (\beta_1^* - \gamma v_1)' \bar{k}_0 + \sum_{t=1}^{T} (\gamma^t p_t - \mu_t^*)' y_t - \sum_{t=1}^{T} \epsilon_t^*.
\end{aligned} \tag{A.8}
$$

The second line of (A.8) follows from (A.4) with $t = 1$. If a firm maximizes $V(\bar{k}_0)$ given p_t, marginal costs are identical to output prices: $\mu_t^* = \gamma^t p_t$. Equation (A.8) is thus reduced to $V(\bar{k}_0) = (\beta_1^* - \gamma v_1)' \bar{k}_0 - \sum \epsilon_t^*$ and $\nabla V(\bar{k}_0) = \beta_1^* - \gamma v_1$. When $\bar{k}_0$ is a scalar, $\nabla V(\bar{k}_0)/u_0$ is referred to as Tobin's marginal q in the literature. On account of discount and deterioration, u_0 may be approximated by $\gamma(1-D)u_1 = -\gamma v_1$ unless a large distortion in the acquisition price occurs in the 1st period. Consequently, Tobin's marginal q is given by $\beta_1^*/u_0 + 1$ in the case of a single quasi-fixed input.

Further, if the technology is linearly homogenous in (x_t, k_{t-1}, k_t, y_t), ϵ_t^* vanishes, and the value function becomes $V(\bar{k}_0) = (\beta_1^* - \gamma v_1)' \bar{k}_0$, implying that the value of a firm is equal to the weighted sum of the shadow values of quasi-fixed inputs. This is a well-known proposition shown by Wildasin (1984). If $\bar{k}_0$ is a scalar, Tobin's average q, $V(\bar{k}_0)/(u_0 \bar{k}_0)$, is equal to Tobin's marginal q, $\beta_1^*/u_0 + 1$. This is also well known as Hayashi's theorem (Hayashi, 1982).

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Notes

1. For example, recent regulatory reforms for public utility industries in many countries aims to improve their efficiency in investment as well as in operation. This requires benchmarking methods to identify the best practice against which the relative performance of utilities is measured. Our procedure is practicable for this purpose, while the conventional DEA is inapplicable to measuring efficiency in investment. See Jamsab and Pollitt (2001) for a survey of benchmarking methods currently employed.
2. It is supposed that variable inputs as well as quasi-fixed inputs at the beginning of the period contribute to the quasi-fixed inputs at the end of the period. This implies that fuel and labor are used in the installation of the quasi-fixed inputs. For example, in electric utility firms by which we illustrate the dynamic DEA later, a substantial number of employees work in the planning section for constructing generation, transmission and distribution facilities, and fuels are consumed in the trial operation of new power plants under inspection before going on stream.
3. Jaenicke (2000) applies the basic dynamic technology for analyzing the rotation effect in crop production.
4. One may choose alternative boundary conditions if necessary. For example, a two-point boundary values problem is easily handled by just setting $k_T = \bar{k}_T$.

5. The original proof of this proposition goes back to Afriat (1972). This property is used by Banker, Charnes and Cooper (1984) to postulate the DEA. Based on a similar argument, Varian (1984) proposes a method for placing an empirical inner limit on the true production possibility set.
6. The expected values conditioned on information available to a firm at each period may be obtained from the time series analysis. However, such an issue is beyond the scope of this paper.
7. Furthermore, we show in the appendix that β_1^* is an essential component of Tobin's marginal q.
8. Since $\bar{k}_0$ is given, v_1 is not relevant to cost minimization. The definition of v_1 will be discussed in Section B of the appendix.
9. See equations (A.6) and (A.7) in Section B of the appendix.
10. Such restrictions are called the assurance region (AR) in DEA literature. See Thompson et al. (1990) for details on the AR approach.
11. A vast literature has reported the fixity of capital in the electric utility industry. For Japanese electric utilities, see Nemoto, Madono and Nakanishi (1993).
12. We owe this idea to Prucha and Nadiri (1996).
13. The objective of equation (4) must be modified because acquisition costs of quasi-fixed inputs now arise from $\tilde{k}_t$ and not k_t. A firm is supposed to choose $\tilde{k}_t$.

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